Course Outline

- Introduction to Sage
  - overview of the software

- Sage demonstration
  - The Sage notebook
  - Getting help
  - Interfaces
  - Matrices
  - Calculus

- Basic programming in Sage
  - Python lists
Sage is a *distribution* of open-source software.
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Software included with Sage:

<table>
<thead>
<tr>
<th>Software</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS</td>
<td>Automatically Tuned Linear Algebra Software</td>
</tr>
<tr>
<td>BLAS</td>
<td>Basic Fortan 77 linear algebra routines</td>
</tr>
<tr>
<td>Bzip2</td>
<td>High-quality data compressor</td>
</tr>
<tr>
<td>Cddlib</td>
<td>Double Description Method of Motzkin</td>
</tr>
<tr>
<td>Common Lisp</td>
<td>Multi-paradigm and general-purpose programming lang.</td>
</tr>
<tr>
<td>CVXOPT</td>
<td>Convex optimization, linear programming, least squares</td>
</tr>
<tr>
<td>Cython</td>
<td>C-Extensions for Python</td>
</tr>
<tr>
<td>F2c</td>
<td>Converts Fortran 77 to C code</td>
</tr>
<tr>
<td>Flint</td>
<td>Fast Library for Number Theory</td>
</tr>
<tr>
<td>FpLLL</td>
<td>Euclidian lattice reduction</td>
</tr>
<tr>
<td>FreeType</td>
<td>A Free, High-Quality, and Portable Font Engine</td>
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<td>Open source Fortran 95 compiler</td>
</tr>
<tr>
<td>GAP</td>
<td>Groups, Algorithms, Programming</td>
</tr>
<tr>
<td>GD</td>
<td>Dynamic graphics generation tool</td>
</tr>
<tr>
<td>Genus2reduction</td>
<td>Curve data computation</td>
</tr>
<tr>
<td>Gfan</td>
<td>Gröbner fans and tropical varieties</td>
</tr>
<tr>
<td>Givaro</td>
<td>C++ library for arithmetic and algebra</td>
</tr>
<tr>
<td>GMP</td>
<td>GNU Multiple Precision Arithmetic Library</td>
</tr>
<tr>
<td>GMP-ECM</td>
<td>Elliptic Curve Method for Integer Factorization</td>
</tr>
<tr>
<td>GNU TLS</td>
<td>Secure networking</td>
</tr>
<tr>
<td>GSL</td>
<td>Gnu Scientific Library</td>
</tr>
<tr>
<td>JsMath</td>
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<td>IML</td>
<td>Integer Matrix Library</td>
</tr>
<tr>
<td>IPython</td>
<td>Interactive Python shell</td>
</tr>
<tr>
<td>LAPACK</td>
<td>Fortan 77 linear algebra library</td>
</tr>
<tr>
<td>Lcalc</td>
<td>L-functions calculator</td>
</tr>
<tr>
<td>Libgcrypt</td>
<td>General purpose cryptographic library</td>
</tr>
<tr>
<td>Libgpg-error</td>
<td>Common error values for GnuPG components</td>
</tr>
<tr>
<td>Linbox</td>
<td>C++ linear algebra library</td>
</tr>
<tr>
<td>Matplotlib</td>
<td>Python plotting library</td>
</tr>
<tr>
<td>Maxima</td>
<td>Computer algebra system</td>
</tr>
<tr>
<td>Mercurial</td>
<td>Revision control system</td>
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<tr>
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</tr>
<tr>
<td>MPFR</td>
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</tr>
<tr>
<td>ECLib</td>
<td>Cremona’s Programs for Elliptic curves</td>
</tr>
<tr>
<td>NetworkX</td>
<td>Graph theory</td>
</tr>
<tr>
<td>NTL</td>
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</tr>
<tr>
<td>Numpy</td>
<td>Numerical linear algebra</td>
</tr>
<tr>
<td>OpenCDK</td>
<td>Open Crypto Development Kit</td>
</tr>
<tr>
<td>PALP</td>
<td>A Package for Analyzing Lattice Polytopes</td>
</tr>
<tr>
<td>PARI/GP</td>
<td>Number theory calculator</td>
</tr>
<tr>
<td>Pexpect</td>
<td>Pseudo-tty control for Python</td>
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<tr>
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</tr>
<tr>
<td>Python</td>
<td>Interpreted language</td>
</tr>
<tr>
<td>Qd</td>
<td>Quad-double/Double-double Computation Package</td>
</tr>
<tr>
<td>R</td>
<td>Statistical Computing</td>
</tr>
<tr>
<td>Readline</td>
<td>Line-editing</td>
</tr>
<tr>
<td>Rpy</td>
<td>Python interface to R</td>
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<td>Python library for scientific computation</td>
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<td>Singular</td>
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Sage is a \textit{distribution} of open-source software.

Software included with Sage:

- \textbf{Sympow}: L-function calculator
- \textbf{Symmetrica}: Representation theory
- \textbf{Sympy}: Python library for symbolic computation
- \textbf{Tachyon}: Lightweight 3d ray tracer
- \textbf{Termcap}: For writing portable text mode applications
- \textbf{Twisted}: Python networking library
- \textbf{Weave}: Tools for including C/C++ code within Python
- \textbf{Zlib}: Data compression library
- \textbf{ZODB}: Object-oriented database

Plus additional optional packages
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Sage is a distribution of *mathematics* software.

*Sage’s mission:* “Creating a viable, free, open-source alternative to Magma™, Maple™, Mathematica™, and Matlab™.”
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“You can read Sylow’s Theorem and its proof in Huppert’s book in the library . . . then you can use Sylow’s Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly . . . .
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(started GAP in 1986)
Sage is a distribution of *free, open-source* software.

You have the freedom:

- to run the program, for any purpose.
- to study how the program works, and adapt it to your needs.
- to redistribute copies so you can help your neighbour.
- to improve the program, and release your improvements to the public, so that the whole community benefits.
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Also, you don’t have to pay for it.
The Sage programming language is Python

Python is a powerful, modern, interpreted programming-language.
The Sage programming language is **Python**

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- *Interpreted* means it works like Maple or Mathematica.

```python
python: x = 17
python: x
17
python: x**2
289
```
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  ```

- It’s *easy to learn*. Lots of free documentation.

  
  - [http://diveintopython.org/](http://diveintopython.org/)
  - [http://docs.python.org/tut/](http://docs.python.org/tut/)
The Sage programming language is **Python**

It's easy to read and write.

\[
x \in \{0, 1, \ldots, 9\} \text{ and } x \text{ is odd}
\]

Python: 

```python
[17*x for x in range(0,10) if x%2 == 1]
```

Lots of Python libraries: databases, graphics, networking, . . .

It is easy to use C/C++ libraries from within Python.

Cython: Python code → compiled C code.
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The Sage programming language is **Python**

“Google has made no secret of the fact they use Python a lot for a number of internal projects. Even knowing that, once I was an employee, I was amazed at how much Python code there actually is in the Google source code system.”

— Guido van Rossum  
(creator of Python)
Several ways to use Sage

- A library for Python scripts.

```python
#!/usr/bin/env sage -python
import sys
from sage.all import *
```
Several ways to use Sage

- Command line interface.

```sage
sage: 17^2
289
```
Several ways to use Sage

- Graphical notebook: try it online at www.sagenb.org.
Sage plays well with \LaTeX

\LaTeX{} input:

\begin{verbatim}
\begin{sagesilent}
  var('s t')
  f = t^2*e^t-sin(t)
\end{sagesilent}

Let $f(t)=\sage{f}$. Then the Laplace transform of $f$ is: $\sage{f.laplace(t,s)}$.
\end{verbatim}

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\LaTeX output:

Let $f(t) = t^2 e^t - \sin(t)$. Then the Laplace transform of $f$ is: $\frac{2}{(s-1)^3} - \frac{1}{s^2+1}$. 
Sage plays well with \LaTeX

\LaTeX\ input:

% Here is an example of a tree:
% \sageplot{Graph({0:[1,2,3], 2:[5]}).plot()}

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Here is an example of a tree:
Sage plays well with \LaTeX

\LaTeX\ input:

\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}
Sage plays well with \textsc{\LaTeX}

\textsc{\LaTeX} input:

\sageplot{plot(-x^3+3*x^2+7*x-4,-5,5)}

\textsc{\LaTeX} output:
Sage plays well with \LaTeX

\LaTeX\ input:

\begin{sagesilent}
    t6 = Tachyon(camera_center=(0,-4,1), xres = 800, yres = 600, \raydepth = 12, aspectratio=.75, antialiasing = True)
    t6.light((0.02,0.012,0.001), 0.01, (1,0,0))
    t6.light((0,0,10), 0.01, (0,0,1))
    t6.texture('s', color = (.8,1,1), opacity = .9, specular = .95, \diffuse = .3, ambient = 0.05)
    t6.texture('p', color = (0,0,1), opacity = 1, specular = .2)
    t6.sphere((-1,-.57735,-0.7071),1,'s')
    t6.sphere((1,-.57735,-0.7071),1,'s')
    t6.sphere((0,1.15465,-0.7071),1,'s')
    t6.sphere((0,0,0.9259),1,'s')
    t6.plane((0,0,-1.9259),(0,0,1),'p')
\end{sagesilent}
\sageplot{t6}
Sage plays well with \LaTeX

\LaTeX\ output:
The Sage community

- Many people have contributed to Sage (directly & indirectly).
- There are several mailing lists.
  - [http://www.sagemath.org](http://www.sagemath.org)
- IRC: #sage-devel on freenode.org.
- Developers are very friendly and helpful.
Let’s use Sage